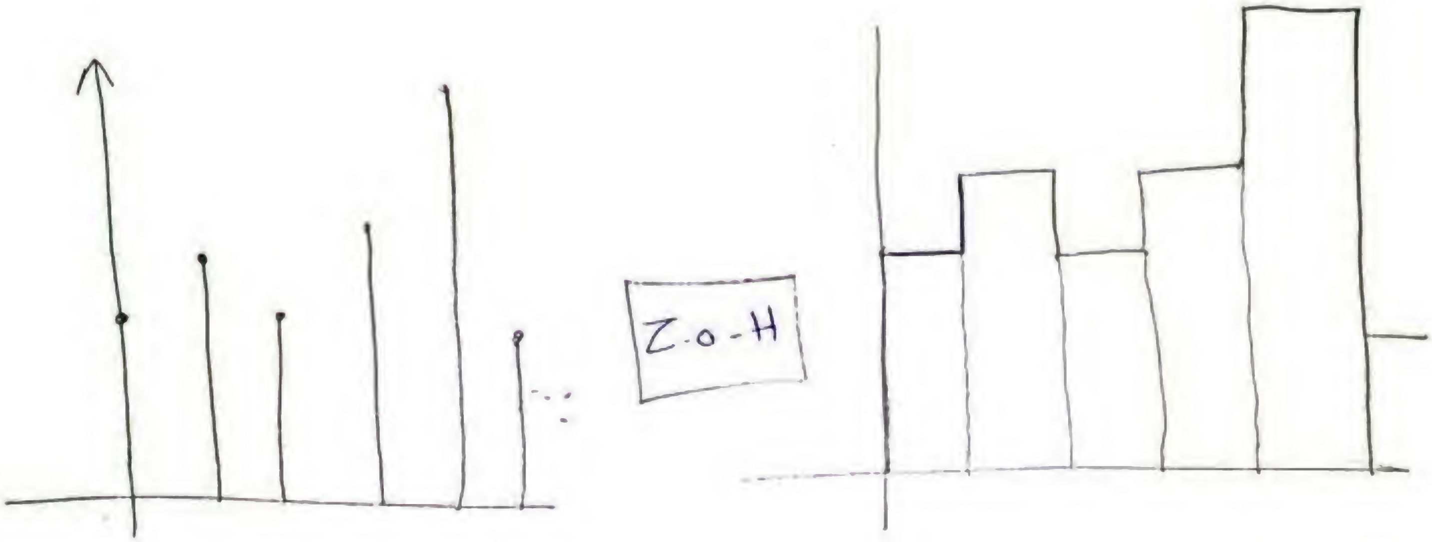
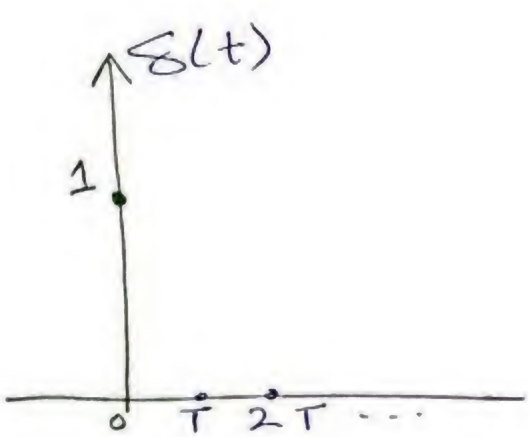
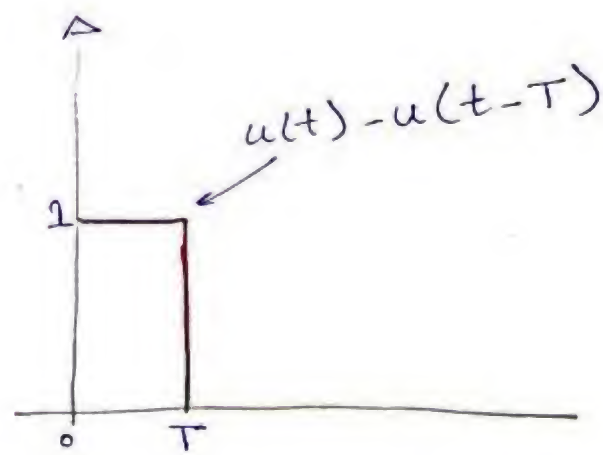
Digital control
Lec 2

The T.F of Z.O.H







T.F for Z.O.H $\Rightarrow G_{Z.O.H}(s) = G_h(s) = \dfrac{O/P(s)}{I/P(s)}$

L.T

$$= \frac{\mathcal{L}.\left(u(t)-u(t-T)\right)}{\mathcal{L}.\left(\delta(t)\right)}$$

$$= \frac{\frac{1}{s} - \frac{e^{-Ts}}{s}}{1} = \frac{1}{s}\left(1 - e^{-Ts}\right)$$

Z.o.H = zero order hold

$$\underset{Z.o.H}{G(s)} = \frac{1 - e^{-Ts}}{s}$$

Z.T $G_h(s) \cdot G_1(s)$

$$Z\left[\frac{1 - e^{-Ts}}{s} \cdot G_1(s)\right] = (1 - z^{-1}) \cdot Z\left[\frac{G_1(s)}{s}\right]$$

* The zero order hold gain = 1

$$Z \cdot \left[G_h(s)\right] = Z\left[\frac{1 - e^{-Ts}}{s}\right]$$

$$= (1 - z^{-1}) \cdot Z\left[\frac{1}{s}\right]$$

$$= (1 - z^{-1})\left(\frac{z}{z-1}\right) = \frac{z-1}{z} * \frac{z}{z-1}$$

$$= 1$$

Pulse T.F

→ The discrete or digital T.F is called Pulse T.F.

$$\text{Pulse T.F} = \frac{C^*(s)}{R^*(s)} = \frac{C(z)}{R(z)}$$

① Pulse T.F from difference equations:-

For ex:

$$y(K-1) + 2y(K) + 3y(K-2) = r(K) \quad \downarrow Z.T$$

Find Pulse $T.F = \dfrac{Y(z)}{R(z)}$

$$z^{-1}Y(z) + 2Y(z) + 3z^{-2}Y(z) = R(z)$$

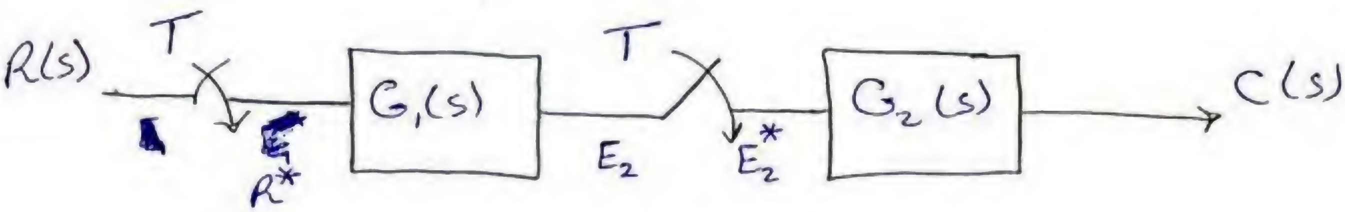
$$(3z^{-2} + z^{-1} + 2)\,Y(z) = R(z)$$

$$\frac{Y(z)}{R(z)} = \frac{1}{3z^{-2} + z^{-1} + 2} = \frac{z^2}{2z^2 + z + 3}$$

$$\boxed{\frac{Y(z)}{R(z)} = \frac{z^2}{2z^2 + z + 3}}$$

2 Pulse T.F from block diagram:-

a) open loop system :-



ـــه هنسمى دخل و خرج ال (Sampler) زي E_2 ، E_2^*

$$C(s) = G_2(s)\, E_2^* \qquad (1)$$

$$E_2 = G_1(s)\, R^* \qquad (2)$$

ـــه نقوم بعمل (staring) للمعادلتين

$$C^*(s) = G_2^*\, E_2^* \qquad (3)$$

$$E_2^* = G_1^*(s)\, R^* \qquad (4)$$

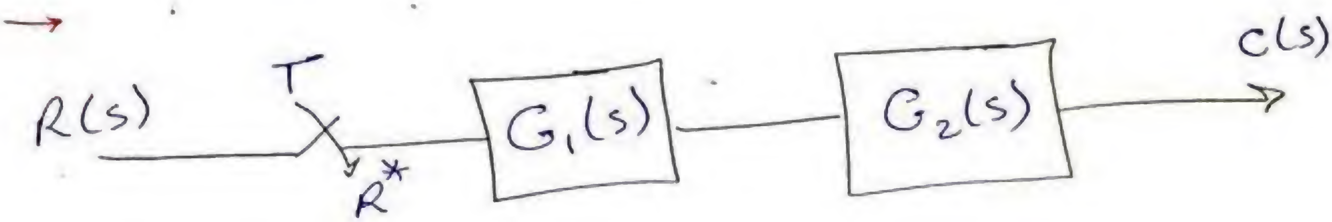
From (4) in (3)

$$C^*(s) = G_1^*(s) \cdot G_2^*(s) \cdot R^*$$

$$\boxed{\frac{C^*(s)}{R^*(s)} = G_1^*(s)\, G_2^*(s)}$$ ← Pulse TF

$$\boxed{\frac{C(z)}{R(z)} = G_1(z) \cdot G_2(z)}$$ ← pulse T.F

→



$$C(s) = G_1(s) \cdot G_2(s) \cdot R^*(s)$$

staring

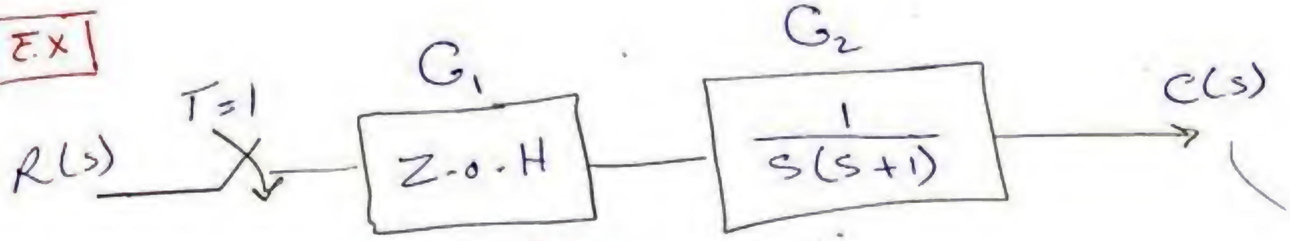
$$C^*(s) = G_1^*(s) \cdot G_2^*(s) \cdot R^*(s)$$

$$\boxed{\frac{C^*(s)}{R^*(s)} = G_1^*(s) \cdot G_2^*(s)}$$

Pulse T.F

$$\boxed{\frac{C(z)}{R(z)} = G_1(z) \cdot G_2(z)}$$

E.X



1) Find $T.F = \frac{C(z)}{R(z)}$

2) Find the unit step response

$$\frac{C(z)}{R(z)} = \overline{G_1 G_2}(z)$$

$$= \mathcal{Z}\left(\frac{1-e^{-Ts}}{s^2(s+1)}\right) = (1-z^{-1})\,\mathcal{Z}\left[\frac{1}{s^2(s+1)}\right]$$

$$= \frac{z-1}{z}\cdot \mathcal{Z}\left[\frac{A_1}{s^2} + \frac{A_2}{s} + \frac{A_3}{s+1}\right]$$

$$A_1 = 1, \quad A_3 = 1$$

$$s=1 \Rightarrow \frac{1}{2} = A_1 + A_2 + \frac{A_3}{2} \Rightarrow A_2 = -1$$

$$\frac{C(z)}{R(z)} = \left(\frac{z-1}{z}\right)\cdot \mathcal{Z}\left[\underset{\substack{\uparrow\\ t}}{\frac{1}{s^2}} - \underset{\substack{\uparrow\\ 1}}{\frac{1}{s}} + \underset{\substack{\uparrow\\ e^{-t}}}{\frac{1}{s+1}}\right]$$

$$= \left(\frac{z-1}{z}\right)\left[\frac{Tz}{(z-1)^2} - \frac{z}{z-1} + \frac{z}{z-e^{-T}}\right]$$

$$= \left(\frac{z-1}{\cancel{z}}\right)\left[\frac{1}{(z-1)^2} - \frac{1}{z-1} + \frac{1}{z-e^{-1}}\right]$$

$$= \frac{1}{Z-1} - 1 + \frac{Z-1}{Z-e^{-1}}$$

$$e^{-1} \simeq 0.368$$

$$\frac{C(Z)}{R(Z)} = \frac{(Z-0.368) - (Z-1)(Z-0.368) + (Z-1)^2}{(Z-1)(Z-0.368)}$$

$$\boxed{\frac{C(Z)}{R(Z)} = \frac{0.368Z + 0.264}{(Z-1)(Z-0.368)}}$$

b $\rightarrow$ unit step response

$$r(t) = 1 \Rightarrow R(Z) = \frac{Z}{Z-1}$$

$$C(Z) = \left[\frac{0.368Z + 0.264}{(Z-1)(Z-0.368)}\right] \cdot R(Z)$$

$$= \frac{0.368Z + 0.264}{(Z-1)(Z-0.368)} * \frac{Z}{Z-1}$$

$$C(Z) = \frac{(0.368Z + 0.264)Z}{(Z-1)^2(Z-0.368)} \Big\Downarrow Z^{-1}\text{-T}$$

$$c(z) = z\left[\frac{0.368z + 0.264}{(z-1)^2 (z-0.368)}\right]$$

$\Downarrow$ using P.F

$$= z\left[\frac{A_1}{(z-1)^2} + \frac{A_2}{z-1} + \frac{A_3}{z-0.368}\right]$$

$A_1 = 1,\ A_2 = -1,\ A_3 = 1$

$$c(z) = z\left[\frac{1}{(z-1)^2} - \frac{1}{z-1} + \frac{1}{z-0.368}\right]$$

$$= \frac{z}{(z-1)^2} - \frac{z}{z-1} + \frac{z}{z-0.368}$$

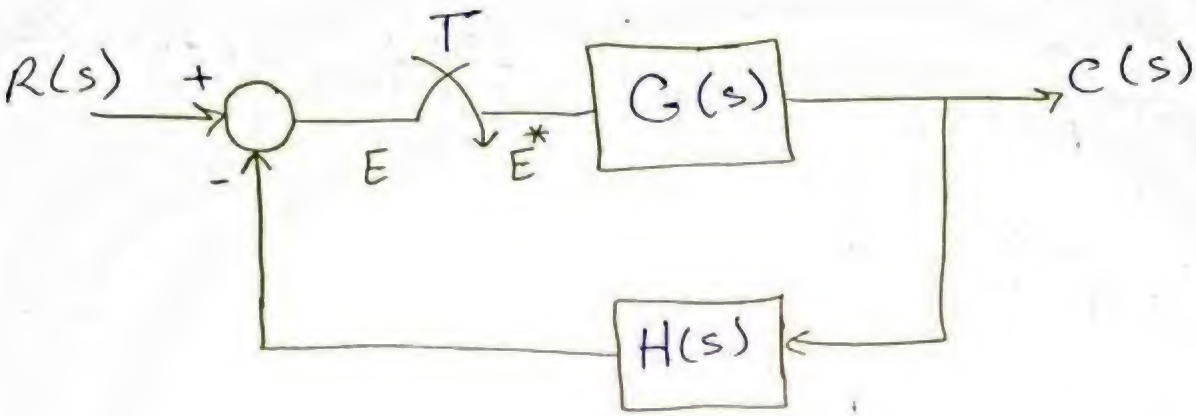
$\Downarrow z^{-1}.T$

$$\boxed{c(k) = k - u(k) + (0.368)^k}$$

↳ unit step response

b) closed loop system

hint: from ways to find pulse T.F from block diagram



1) تسمية لدخل وخرج ال (sampler).

2) نحسب معادلة الخرج الرئيسي C(s) بالإضافة لمعادلات ال i/ps الخاصة بال (sampler).

3) قبل عمل (staring) إذا تواجدت تعويضات نقوم بعملها.

4) نعمل عملية (staring) للمعادلات.

$$\boxed{C(s) = G(s)\, E^*(s)} \longrightarrow (1)$$

$$E(s) = R(s) - H(s) \cdot C(s)$$

$$\cancel{= R(s) - GH(s)\, E(s)}$$

$$\boxed{E(s) = R(s) - GH(s) \cdot E^*(s)} \longrightarrow (2)$$

staring for ① , ②

$$C^*(s) = G^*(s) \cdot E^*(s) \longrightarrow (3)$$

$$E^*(s) = R^*(s) - \overline{GH(s)}^* \cdot E^*(s) \longrightarrow (4)$$

$$\left(1 + \overline{GH(s)}^*\right) E^*(s) = R^*(s)$$

$$E^*(s) = \frac{R^*(s)}{1 + \overline{GH(s)}^*} \longrightarrow (5)$$

from (5) in (3)

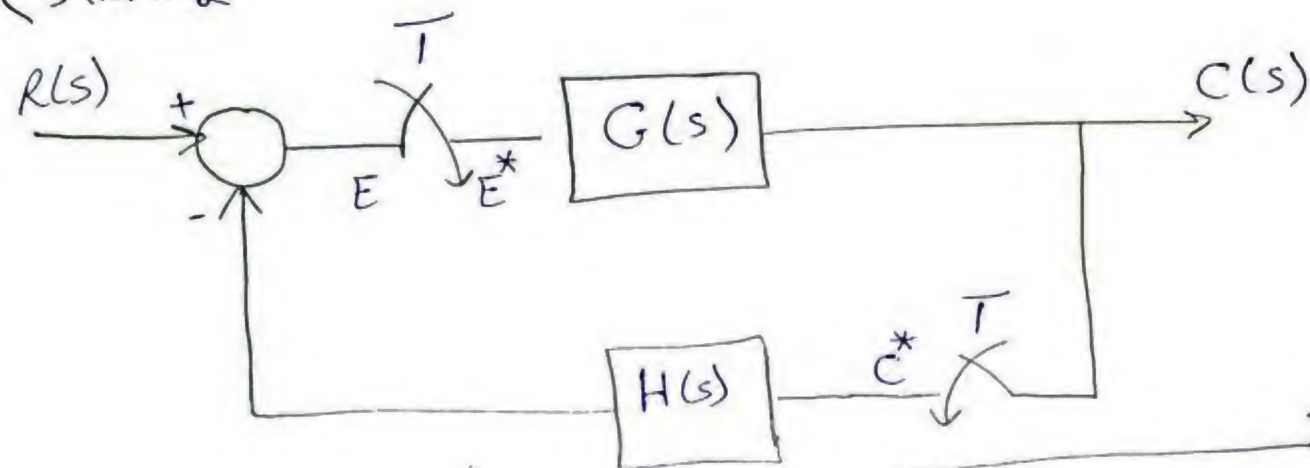
$$C^*(s) = \frac{G^*(s) \cdot R^*(s)}{1 + \overline{GH(s)}^*}$$

$$\boxed{\frac{C^*(s)}{R^*(s)} = \frac{G^*(s)}{1 + \overline{GH(s)}^*}}$$

Pulse T.F

$$\boxed{\frac{C(z)}{R(z)} = \frac{G(z)}{1 + \overline{GH}(z)}}$$

← اذا قمت بعمل (staring) قبل التعويض مش هتعرف
تحسب ال (Pulse T.F) .(لكن فيه حالات بنعوض فيها بعد ال
(staring



$$C(s) = G(s) \cdot E^*(s) \longrightarrow \boxed{C^*(s) = G^*(s) \cdot E^*(s)} \rightarrow (1)$$

$$E = R - H(s) \cdot C^*(s) \longrightarrow E^* = R^* - H^* C^*(s)$$

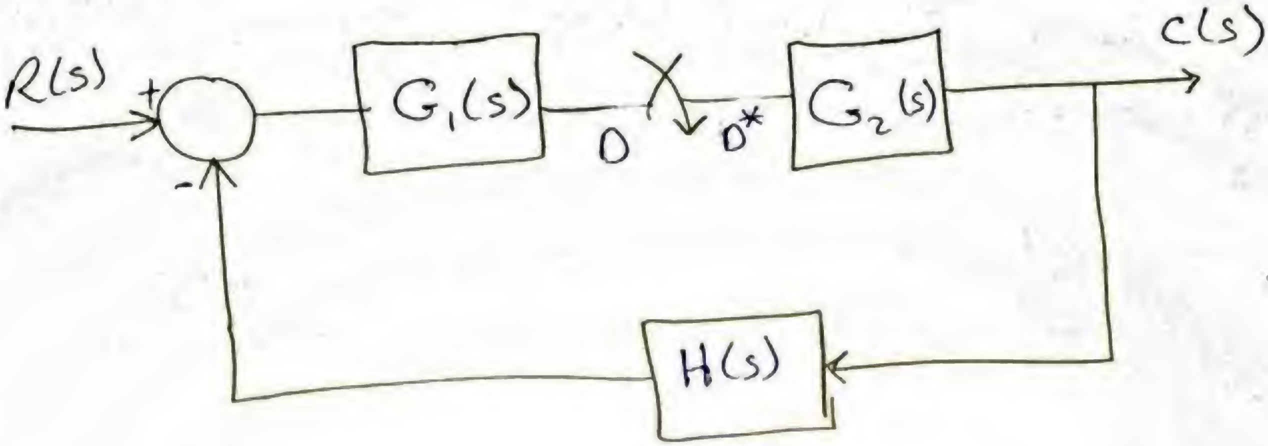
$$E^* = R^* - G^*(s) H^*(s) E^*(s)$$

$$\left(1 + G^*(s) H^*(s)\right) E^* = R^*$$

$$\boxed{E^* = \frac{R^*}{1 + G^*(s) H^*(s)}} \longrightarrow (3)$$

From (3) in (1)

$$\boxed{\frac{C^*}{R^*} = \frac{G^*(s)}{1 + G^*(s) H^*(s)}}$$



$$C(s) = G_2(s) \cdot D^*(s) \longrightarrow (1)$$

$$D(s) = G_1(s)\left[R(s) - C(s) \cdot H(s) \cdot D^*(s)\right]$$

$$D(s) = G_1 R(s) - G_1 G_2 H(s) D^*(s) \longrightarrow (2)$$

staring

$$C^*(s) = G_2^*(s) \cdot D^*(s) \longrightarrow (3)$$

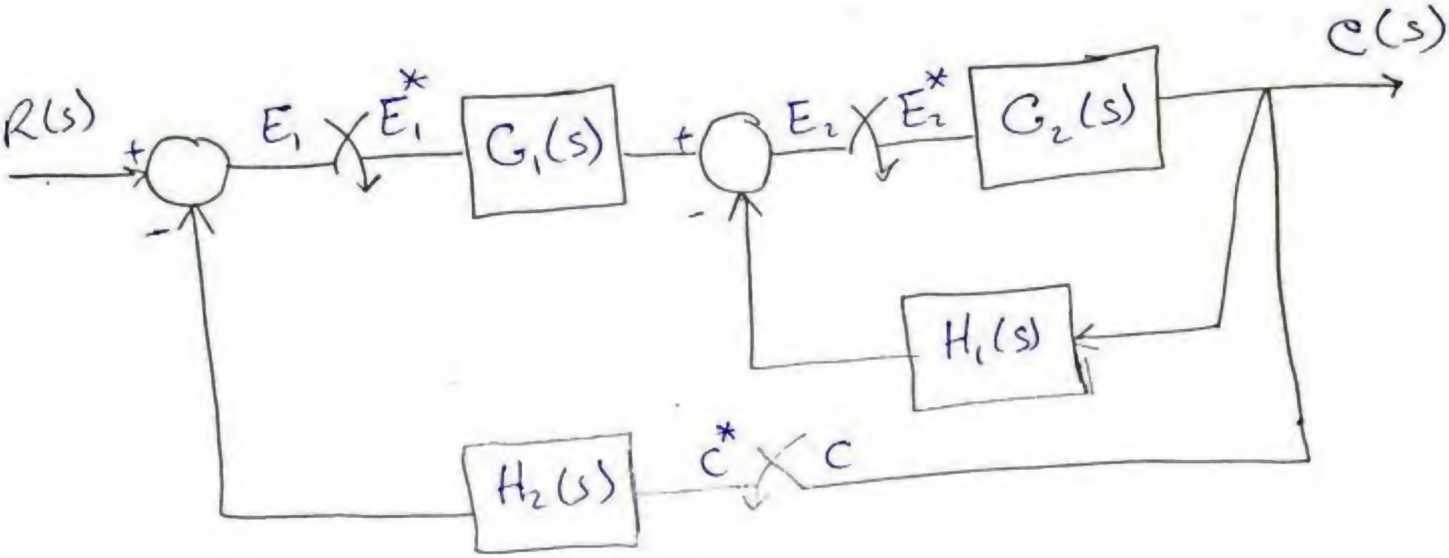
$$D^*(s) = \overline{G_1 R}^*(s) - \overline{G_1 G_2 H}^*(s)\, D^*(s) \longrightarrow (4)$$

↳ we can't find impulse T.F

$$C^*(s) = \frac{G_2^* \cdot \overline{G_1 R}^*(s)}{1 + \overline{G_1 G_2 H}^*(s)}$$

$$\boxed{C(z) = \frac{G_2(z) \cdot \overline{G_1 R}(z)}{1 + \overline{G_1 G_2 H}(z)}}$$

↳ system o/p



ـــ ممكن تتحسب بمجرد النظر

$$\frac{C(z)}{R(z)} = \frac{G_1(z) \cdot G_2(z)}{1 + \underbrace{\overline{G_2 H_1}(z)}_{} + G_1(z) \cdot G_2(z)\, H_2(z)}$$

أول (feedback) على يمين الرسم .

ـــ لكنك مطالب في الامتحان __بالخطوات__ .

$$c(s) = G_2(s) \cdot E_2^*(s) \longrightarrow (1)$$

$$E_2(s) = G_1(s)\, E_1^* - G_2 H_1 E_2^* \longrightarrow (2)$$

$$E_1(s) = R(s) - H_2(s)\, C^*(s) \longrightarrow (3)$$

staring

$$C^*(s) = G_2^*(s) \cdot E_2^*(s) \longrightarrow a$$

$$E_2^*(s) = G_1^*(s)\, E_1^* - \overline{G_2 H_1}^*\, E_2^* \longrightarrow b$$

$$E_1^*(s) = R^*(s) - H_2(s)\, G_2^*(s) \cdot E_2^*(s) \longrightarrow c$$

From c in b

$$E_2^* = G_1^*\left(R^* - H_2^* G_2^* \cdot E_2^*\right) - \overline{G_2 H_1(s)}^*\, E_2^*(s)$$

$$= G_1^* R^* - G_1^* G_2^* H_2^* E_2^* - \overline{G_2 H_1(s)}^*\, E_2^*(s)$$

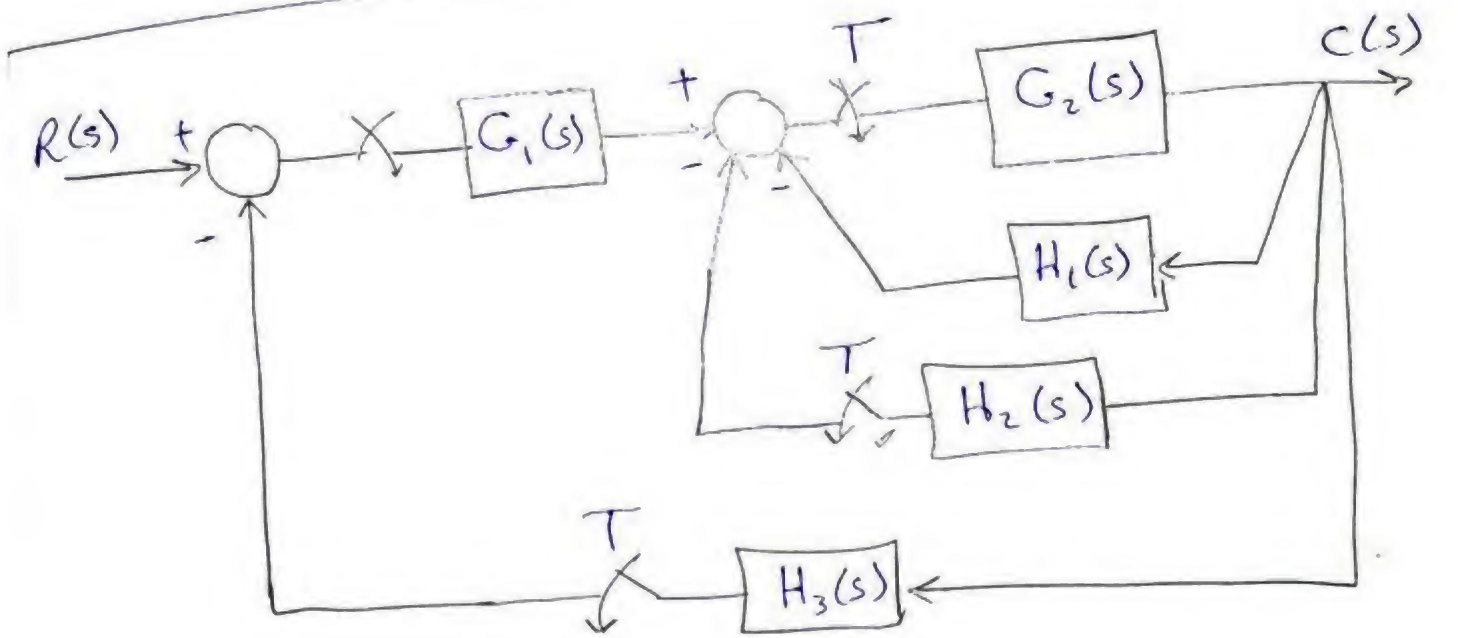
$$\left(1 + \overline{G_2 H_1(s)}^* + G_1^* G_2^* H_2^*\right) E_2^* = G_1^*(s)\, R^*(s)$$

$$\boxed{E_2^* = \frac{G_1^*\, R^*(s)}{1 + \overline{G_2 H_1(s)}^* + G_1^* G_2^* H_2^*}} \longrightarrow d$$

From (d) in (a)

$$\frac{C^*(s)}{R^*(s)} = \frac{G_1^*(s) \cdot G_2^*(s) \cdot}{1 + \overline{G_2 H_1}^*(s) + G_1^* G_2^* H_2^*}$$

$$\frac{C(z)}{R(z)} = \frac{G_1(z) \cdot G_2(z)}{1 + \overline{G_2 H_1}(z) + G_1(z) \cdot G_2(z) H_2(z)}$$



← بالنظر

$$\frac{C(z)}{R(z)} = \frac{G_1(z) \cdot G_2(z)}{1 + \overline{G_2 H_1}(z) + \overline{G_2 H_2}(z) + G_1(z) \cdot \overline{G_2 H_3}(z)}$$

← لو مكان ال (sampler) فى الاتجاه الآخر هيفرق
لأنه هيفصل ما بين آخر حدين فى المقام.